Note Title 9/19/2006

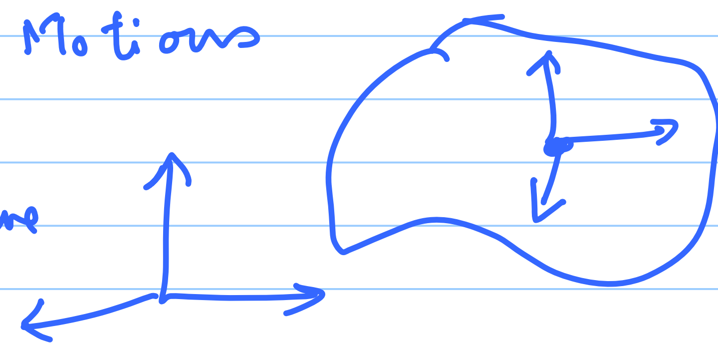
Lecture 5

Basic KINEMATICS / Mechanics :

1. Rep. of rigid body motions
2. ~~friction~~
3. forces / ~~wrenches~~
4. ~~notion of form / force closure~~ possibly later

1) Rigid body Motions

attach a co-ord frame to a rigid body



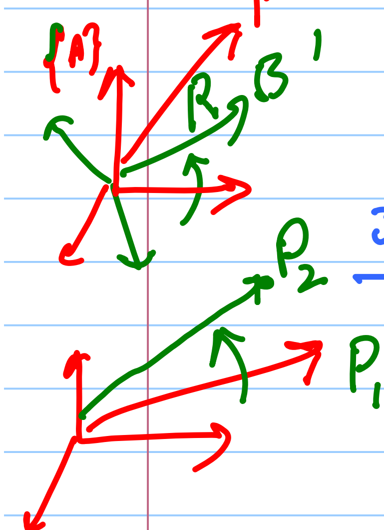
and describe motions of the frame.

$\{Trans, Rot\}$

① Translation $\longrightarrow$ vectors in R^N.

② Rotation $\longrightarrow$ matrices $N \times N$

gen'l rigid body motion $\longrightarrow$ $(Trans, R)$ = $\{$ 3x1 vector, 3x3 matrix $\}$



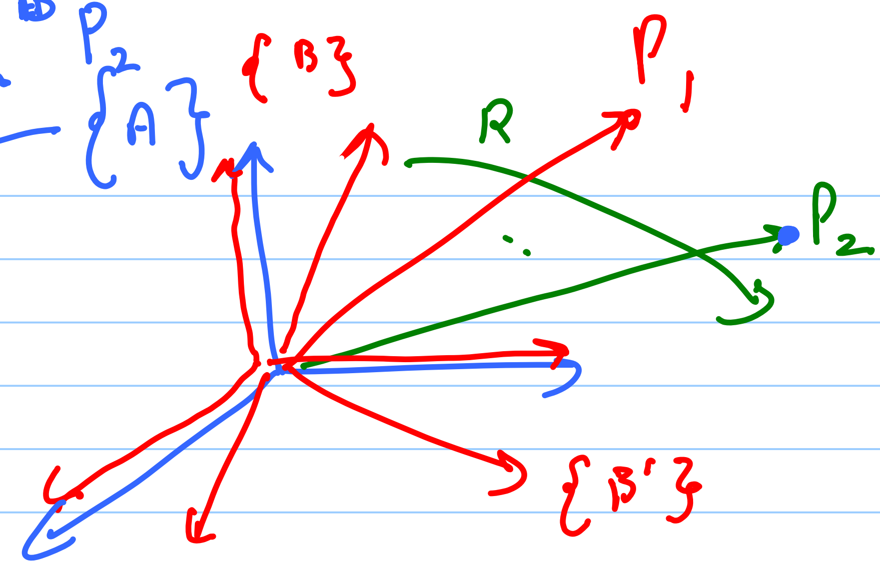
3 interpretation : ${}^{A}_{B'}R = R$

① Trans Rot $\longrightarrow$ used to rep. a frame

② $P_2 = R \cdot P_1$ $\longrightarrow$ " " rep. trans. & rot.

${}^{B'}P_2 = {}^{A}P_1$ ③ $\{B'\} \xrightarrow{{}^{A}_{B'}R} \{A\}$, ${}^{B'}P_2 \to {}^{A}P_2$ $\longrightarrow$ mapping from one frame to other

$$^{A}P_2 = {}^{A}_{B'}R \; {}^{B'}P_2$$



$${}^{A}_{B'}R = \underline{\underline{R}}$$

"orthonormal" matrices SO(N)

$$\begin{array}{cccc} & \text{unit vec 1} & \text{unit vec 2} & \text{unit vec 3} \\ R = (r_{ij}) = & \Bigg(\; \Big| & \Big| & \Big| \; \Bigg) \end{array}$$

$\perp$ $\perp$

$\det(R)$

$|R| = +1$

$r_{11}^2 + r_{21}^2 + r_{31}^2 = 1$

$r_{11} \cdot r_{12} + r_{21} r_{22} + r_{31} r_{32} = 0 \perp$

$3 \times 3 = 9$ elem.

6 orthonormal const.

$\underline{\underline{3}}$ indep. parameters

N - dim : $N \times N$ $\qquad N^2$ elem

$\underline{\underline{\det = +1}}$

$$\left.\begin{array}{ll} {}^{N}C_2 \text{ const.} & \text{mutual ortho.} \\ N \quad " & \text{unit vec.} \end{array}\right\} {}^{N}C_2 + N \text{ const.}$$

$$\underline{M = N^2 - \left[\frac{N(N-1)}{2} + N \right]} \qquad \underline{\underline{\text{ind. elem.}}}$$

$\underline{S}\,\underline{O}(N) \longleftrightarrow$ Rotations in N dim

$\boxed{\mathbb{R}^T = \mathbb{R}^{-1}}$

$N = 3 \Rightarrow M = 9 - [3 + 3] = 3$

$N = 2 \Rightarrow M = 4 - [1 + 2] = 1$

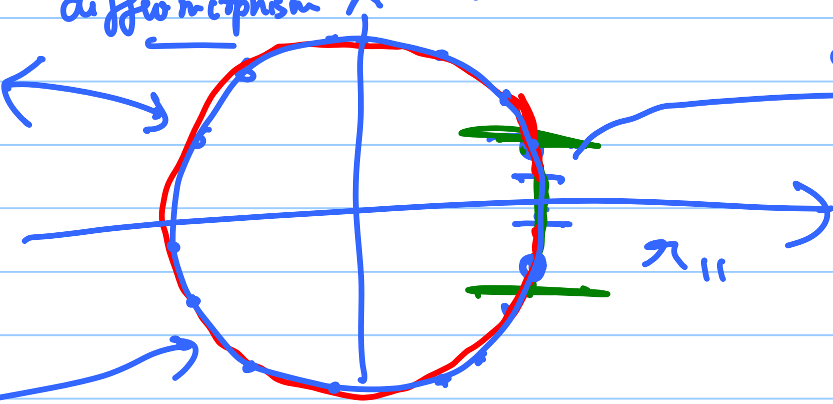
$$\begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \qquad \begin{array}{ll} r_{11}^2 + r_{21}^2 = 1 & r_{11} \cdot r_{12} + \\ r_{21}^2 + r_{22}^2 = 1 & r_{21} \cdot r_{22} = 0 \end{array}$$

$$\underline{SO(2)} = \begin{pmatrix} c\theta & -s\theta \\ s\theta & c\theta \end{pmatrix} = \begin{pmatrix} r_{11} & -r_{21} \\ r_{21} & r_{11} \end{pmatrix}$$

$$r_{11}^2 + r_{21}^2 = 1$$

r_{11}, r_{21}

diffeomorphism



$SO(2)$

$\underline{R^2 \times SO(2)}$

diffeo ✗

Θ $\mathbb{R}^1$

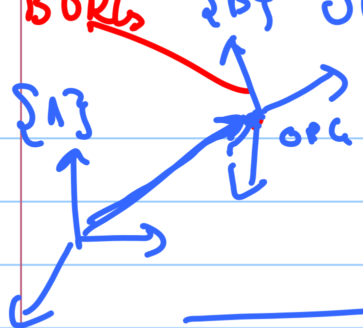
S^2 $\quad S^1$ $\quad$ modulo (2π) $\quad (0, 2\pi)$

$SO(3)$: α, β, γ $\quad$ Euler Angles $\rightarrow$ w.r.t. current axes

$\hat{x}$ $\hat{y}$ $\hat{z}$ $\quad$ fixed Ang $\rightarrow$ " fixed axes

$R_{\hat{k}}(\theta) = (k_x, k_y, k_z), \Theta$

$^{A}P_{BORG}$



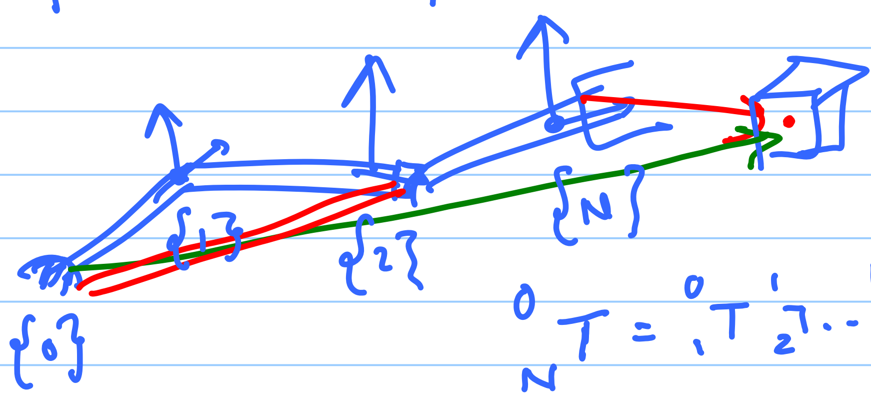
$SO(3) \longleftrightarrow$ 3-dim. manifold

an atlas $SO(3) \xrightarrow{\phi_i} (0,2\pi) \times (0,\ddot{\pi})$??

$(0,2\pi)$

$^{A}_{B}T = \left(\begin{array}{c|c} ^{A}_{B}R & ^{A}P_{BORG} \\ \hline 0 & 1 \end{array}\right)$

from, D-H parameters: To each link assign a frame



$\{i\}$ is attached to link $\{i\}$

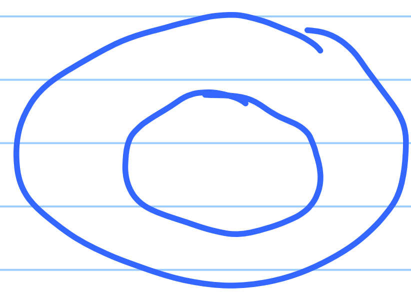
$^{0}_{N}T = {}^{0}_{1}T\, {}^{1}_{2}T \cdots {}^{N-1}_{N}T$ moves with joint $\{i\}$

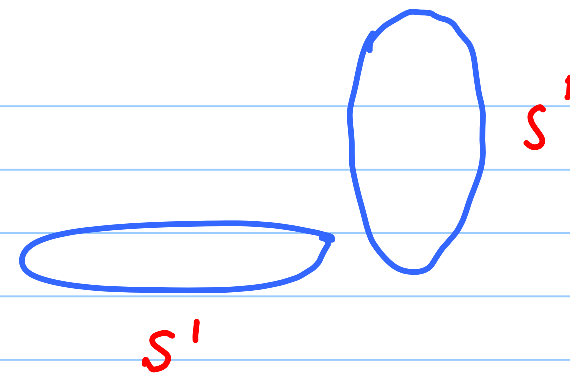
open vs closed chains

RIGID BODY: $R^N \times SO(N) = SE(N)$

arm with all rev. joints : $S^1 \times S^1 \times S^1 \times \cdots S^1$ Torus(N)

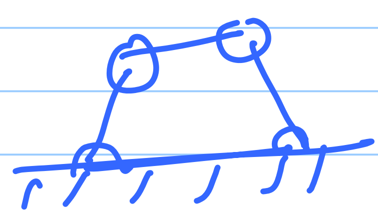
$S^1 \times S^1$





# of ind. parameters : degrees of freedom in the chain

formula for closed chains:



one link is "fixed"

n-1 movable links , N rigid bodies (links)

each joint has $N - f_i$ const.

n joints

Space of ind. par. needed to specify a config of the other c-space

\# of dofs : $N(k-1) - \sum_{i=1}^{n} (N - f_i)$

$= N(k-n-1) + \sum_{i=1}^{n} f_i$

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velocities : inst. motion 2 or 3 dim.

$\downarrow$ vector alg.

$SO(N)$

$\underline{\underline{N \times N}}$

${}^{A}_{B}R \quad R \quad \dot{R}$

$R R^T = I$

$\dot{R} R^T + R \dot{R}^T = 0$

$\Leftrightarrow \underbrace{\dot{R} R^T}_{S} = -R \dot{R}^T$

$p'(t) = R(t) \cdot p$

$\dot{p}'(t) = \dot{R}(t) \cdot p + \underline{\underline{R(t) \cdot \dot{p}}}$

$= \underbrace{\dot{R} R^T} R p + R(t) \dot{p}$

$= S \cdot R p$

$\Leftrightarrow \quad S = -S^T \quad \longleftarrow$ skew symm $\longrightarrow \underline{\underline{\Omega \times RP}}$

for $\longrightarrow \begin{pmatrix} 0 & \Omega_x & \Omega_y \\ & 0 & \\ \vdots & & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{pmatrix}$

3 ind. par.

$S \cdot \vec{P} = \underline{\underline{\vec{\Omega}}} \times \vec{P}$

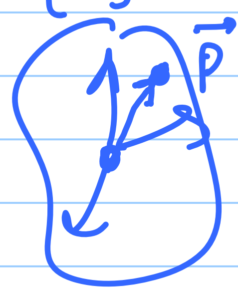
ang. vel. vect.

rate of

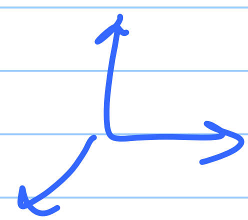
$|\Omega| =$ mag of inst. rot.

$\dfrac{\Omega}{|\Omega|} =$ dir or axis of rot.

{B}

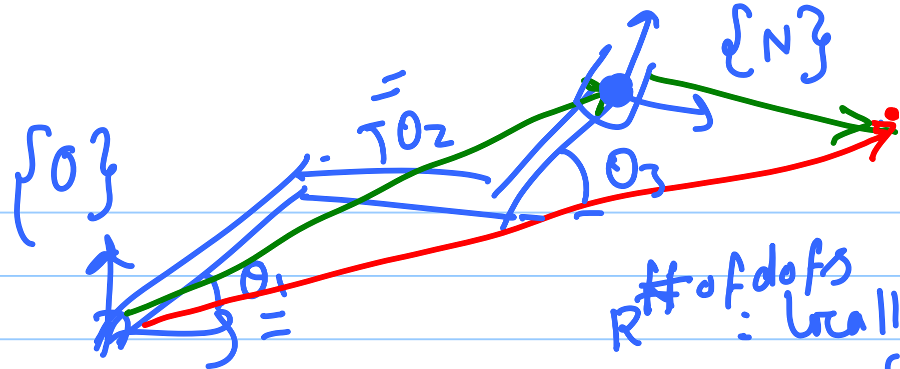


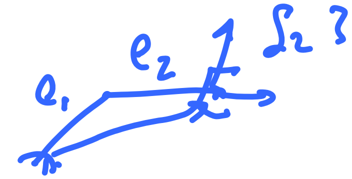
{A}



$\cancel{{}^A\vec{v}_P} \stackrel{?}{=} {}^B v_P$

$= \underline{{}^A v_{BORG}} + \underline{{}^A_B R\, {}^B v_P} + \underline{\underline{{}^A\Omega_B \times {}^A_B R\, {}^B P}}$





$\mathbb{R}^{\#\text{ of dofs}}$: locally

$e_1 c_1 + e_2 c_{12}$

$${}^{0}_{N}T \left[f(\theta_1 \dots \theta_n) \right] \qquad {}^{0}_{2}T = \begin{pmatrix} c_{12} & -s_{12} & 0 & \\ s_{12} & c_{12} & 0 & \\ 0 & 0 & 1 & \end{pmatrix}$$

$$\begin{bmatrix} {}^{0}_{2}R & {}^{0}P_{2ORG} \\ \underline{0} & 1 \end{bmatrix}$$

$e_1 s_1 + e_2 s_{12}$

$\dot{\theta}_1 \xrightarrow{J} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \omega_x \end{pmatrix}$ θ_1

J : differential map

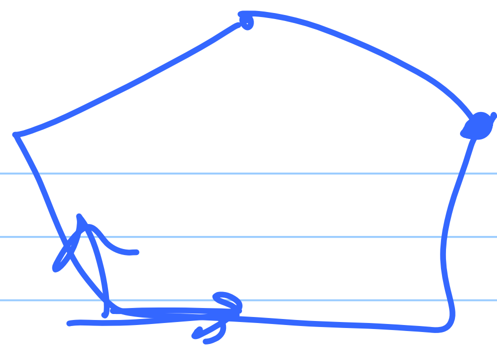
$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} = J \begin{pmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_N \end{pmatrix}$$

$J = 6 \times N$ matrix

$$\begin{pmatrix} \quad \end{pmatrix}$$

$\underline{\dot{x}} = J \dot{q}$

won't cover
how to compute
J for orient.
arm config

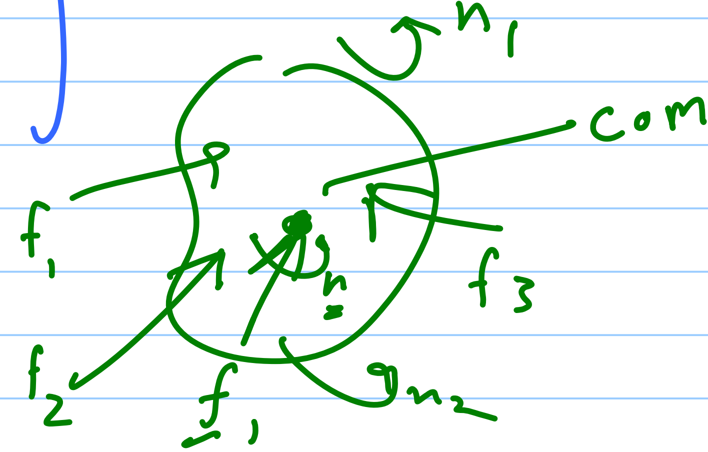


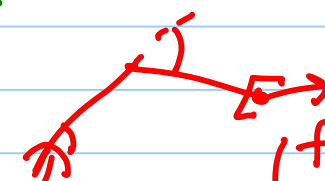
$$\underline{f} = \begin{pmatrix} f_1(x_1, \dots, x_n) \\ f_2(x_1, \dots, x_n) \\ \vdots \\ f_m(x_1, \dots, x_n) \end{pmatrix} \quad R^{n} \to R^{m}$$

$$= \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \cdots & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_m}{\partial x_1} & \cdots & \cdots & \frac{\partial f_m}{\partial x_n} \end{pmatrix}$$

forces $\sum_i$ $\begin{pmatrix} f_x \\ f_y \\ f_3 \\ n_x \\ n_y \\ n_3 \end{pmatrix}$

$\underline{f}, \underline{n}$





$\begin{pmatrix} f_x \\ f_y \end{pmatrix}$

$$\underline{F} = \begin{pmatrix} \underline{f} \\ \underline{n} \end{pmatrix} \text{ : gen. force at end.eff.}$$

Princip. of vir. work

$$\underline{\dot{x}} = J\,\underline{\dot{q}}$$

$$V = \begin{pmatrix} v \\ \omega \end{pmatrix} \leftarrow \text{inst. motion at end eff.}$$

$$\boxed{\begin{aligned} F^T \underline{\dot{x}} &= \tau_c^T \dot{q} \\ \cancel{F^T J \dot{q}} &\cancel{= \tau_c^T \dot{q}} \end{aligned}}$$ virtual work

$$F^T V = \text{work done at end-eff.}$$

$$\Rightarrow \boxed{\tau_c = J^T F}$$

$$V_c = \begin{pmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_N \end{pmatrix} \qquad \tau_c = \begin{pmatrix} \tau_1 \\ \vdots \\ \tau_N \end{pmatrix} \qquad \tau_c^T V_c = \text{work done at joints}$$

$$\Longrightarrow F^T V = \tau_c^T V_c$$

$$V = J V_c \Rightarrow F^T J V_c = \tau_c^T V_c$$

$$\Longrightarrow \boxed{\tau_c = J^T F}$$

Closest example: how the forces are related

wrench / screw → Read Bernie Roth handout
read

friction / friction cones → Mason's paper

spatial rep. ⟶ Read Mantyla's handout

Skipped form closure/force closure will do it later

[Grasp]